On S-Box Reverse-Engineering: from Cryptanalysis to the Big APN Problem

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Boolean Functions and Their Applications

The content of this talk is based on joint works with Biryukov, Canteaut, Duval, Khovratovich and Udovenko, and my PhD thesis.

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Structured?

Was it built using a particular structure ?

S-Box?

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- Typically, $n = m, n \in \{4, 8\}$
- Used by many block ciphers/hash functions/stream ciphers.
- Necessary for the wide trail strategy.

Example

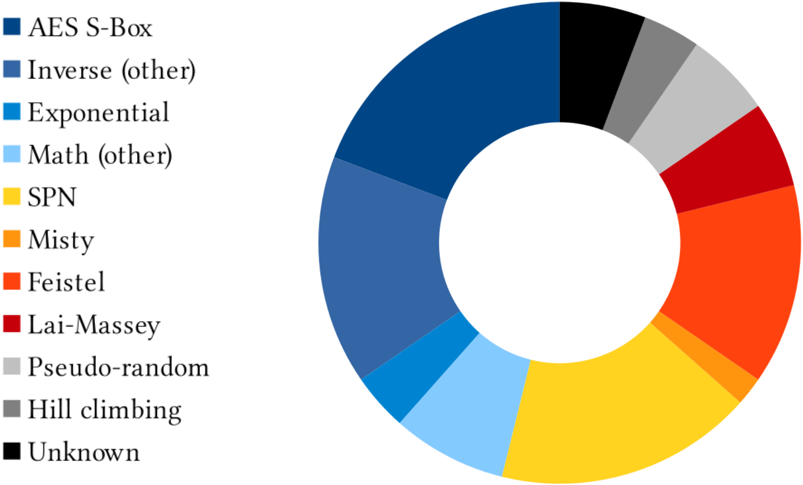
π' = (252, 238, 221, 17, 207, 110, 49, 22, 251, 196, 250, 218, 35, 197, 4, 77, 233, 119, 240, 219, 147, 46, 153, 186, 23, 54, 241. 187, 20, 205, 95, 193, 249, 24, 101, 90, 226, 92, 239, 33, 129, 28, 60, 66, 139, 1, 142, 79, 5, 132, 2, 174, 227, 106, 143, 160, 6, 11, 237, 152, 127, 212, 211, 31, 235, 52, 44, 81, 234, 200, 72, 171, 242, 42, 104, 162, 253, 58, 206, 204, 181, 112, 14, 86, 8, 12, 118, 18, 191, 114, 19, 71, 156, 183, 93, 135, 21, 161, 150, 41, 16, 123, 154, 199, 243, 145, 120, 111, 157, 158, 178, 177, 50, 117, 25, 61, 255, 53, 138, 126, 109, 84, 198, 128, 195, 189, 13, 87, 223, 245, 36, 169, 62, 168, 67, 201, 215, 121, 214, 246, 124, 34, 185, 3, 224, 15, 236, 222, 122, 148, 176, 188, 220, 232, 40, 80, 78, 51, 10, 74, 167, 151, 96, 115, 30, 0, 98, 68, 26, 184, 56, 130, 100, 159, 38, 65, 173, 69, 70, 146, 39, 94, 85, 47, 140, 163, 165, 125, 105, 213, 149, 59, 7, 88, 179, 64, 134, 172, 29, 247, 48, 55, 107, 228, 136, 217, 231, 137, 225, 27, 131, 73, 76, 63, 248, 254, 141, 83, 170, 144, 202, 216, 133, 97, 32, 113, 103, 164, 45, 43, 9, 91, 203, 155, 37, 208, 190, 229, 108, 82, 89, 166, 116, 210, 230, 244, 180, 192, 209, 102, 175, 194, 57, 75, 99, 182).

Screen capture from [GOST, 2015].

S-Box Design

- AES S-Box
- Inverse (other)
- Exponential
- Math (other)
- SPN
- Misty
- Feistel
- Lai-Massey
- Pseudo-random
- Hill climbing
- Unknown

S-Box Design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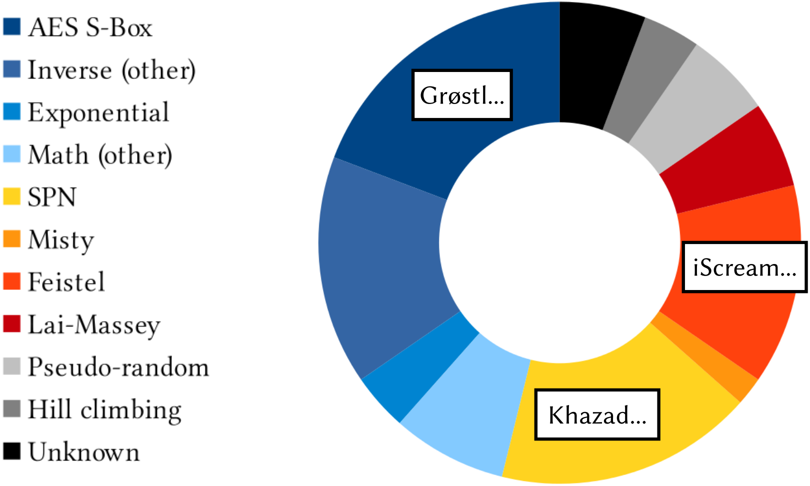
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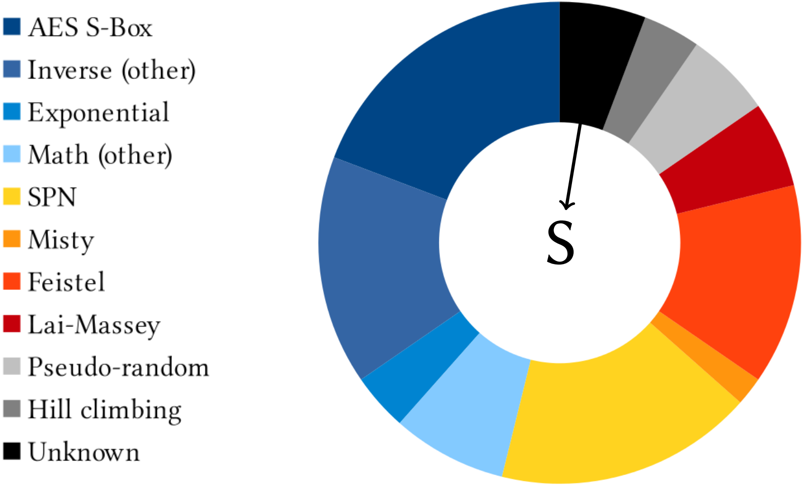
Grøstl...
iScream...
Khazad...

S-Box Reverse-Engineering



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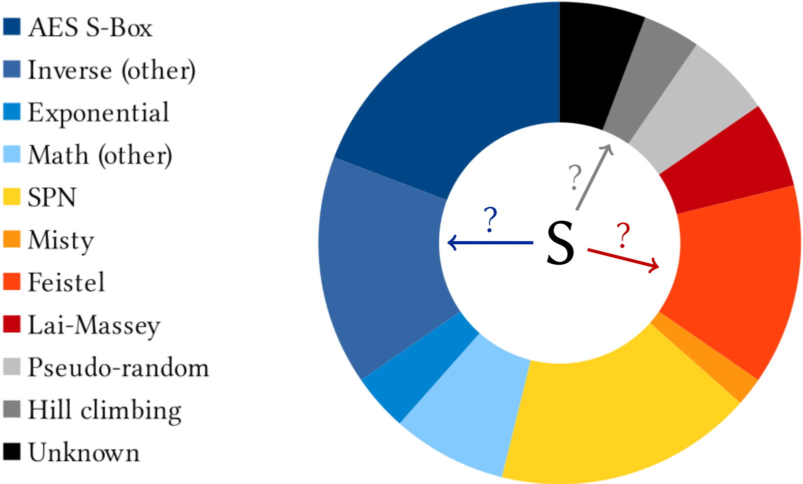

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To keep an advantage in implementation (WB crypto)...
... or an advantage in cryptanalysis (backdoor)?

Outline

Plan of this Section

The Two Tables

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Definition (DDT)

The *Difference Distribution Table* of S is a matrix of size $2^n \times 2^n$ such that

$$\mathrm{DDT}[a, b] \; = \; \#\{x \in \mathbb{F}_2^n \mid S(x \oplus a) \oplus S(x) = b\}.$$

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Definition (LAT)

The *Linear Approximations Table* of S is a matrix of size $2^n \times 2^n$ such that

$$\mathrm{LAT}[a, b] \;=\; \#\{x \in \mathbb{F}_2^n \mid x \cdot a = S(x) \cdot b\} - 2^{n-1} \;=\; \frac{\mathcal{W}_S(a, b)}{2}$$

Coefficient Distribution in the DDT

If an n-bit S-Box is bijective, then its DDT coefficients behave like independent and identically distributed random variables following a Poisson distribution:

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- Always even, ≥ 0
- Typically between 0 and 16 (for $n =$)
- Lower is better.

Coefficient Distribution in the LAT

If an n-bit S-Box is bijective, then its LAT coefficients behave like independent and identically distributed random variables following this distribution:

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- Always even, signed.
- Typically between -40 and 40 (for $n = 8$).
- Lower absolute value is better.

Looking Only at the Maximum

δ	$\log_2 (\Pr [\max(\mathcal{D}) \leq \delta])$
14	-0.006
12	-0.094
10	-1.329
8	-16.148
6	-164.466
4	-1359.530

DDT

ℓ	$\log_2 (\Pr [\max(\mathcal{L}) \leq \ell])$
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LAT

Probability that the maximum coefficient in the DDT/LAT of an 8-bit permutation is at most equal to a certain threshold.

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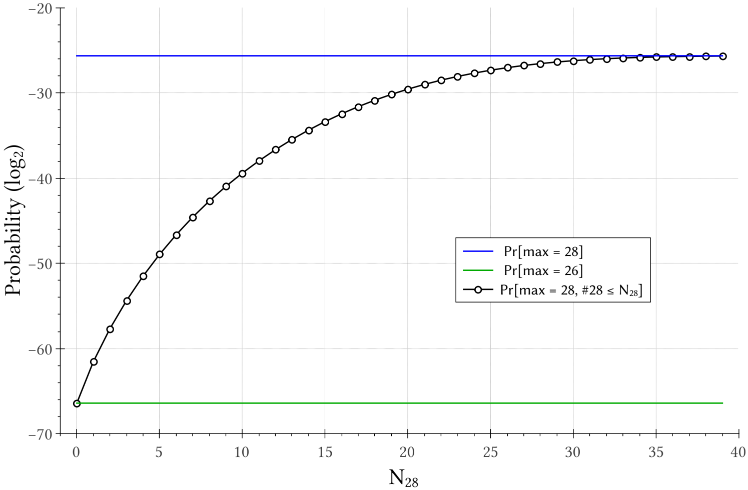
Taking Number of Maximum Values into Account

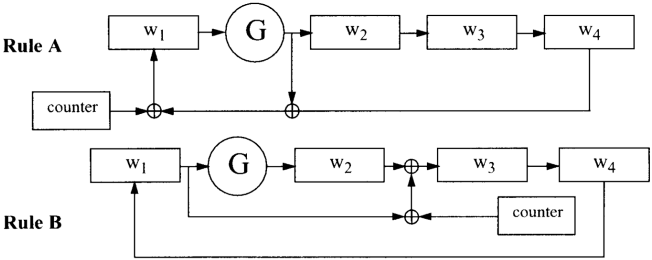


Application of this Analysis?

We applied this method on the S-Box of Skipjack.

What is Skipjack?

Type	Block cipher
Bloc	64 bits
Key	80 bits
Authors	NSA
Publication	1998 (classified at first)



Reverse-Engineering the S-Box of Skipjack

Skipjack uses F, a permutation of $\mathbb{F}_2^8$ with $\max(\text{LAT}) = 28$ and $\#28 = 3$.

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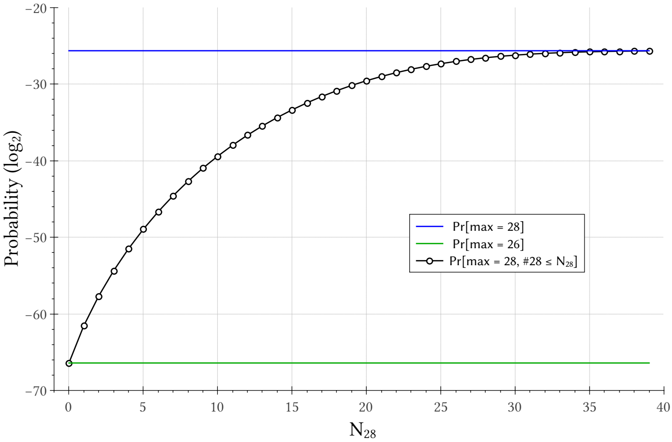

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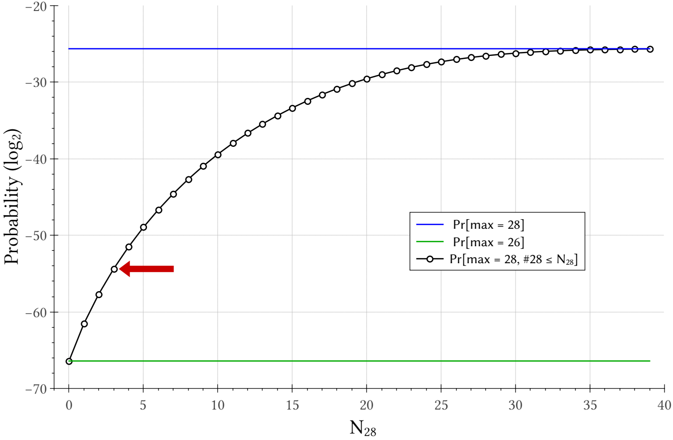

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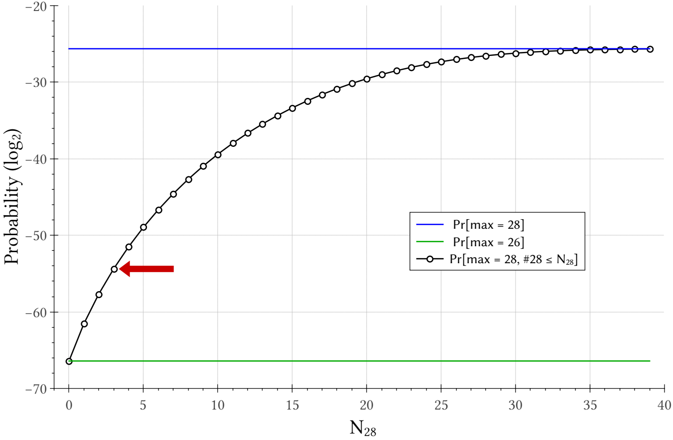

$$\Pr\left[\max(\text{LAT}) = 28 \text{ and } \#28 \leq 3\right] \approx \mathbf{2^{-55}}$$

What Can We Deduce?

- F has not been picked uniformly at random.
- F has not been picked among a feasibly large set of random S-Boxes.
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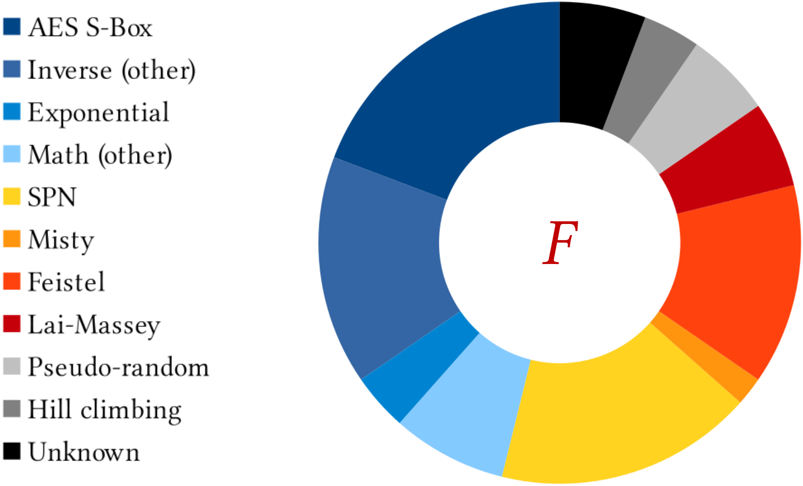
The S-Box of Skipjack was built using a dedicated algorithm.

Conclusion on Skipjack



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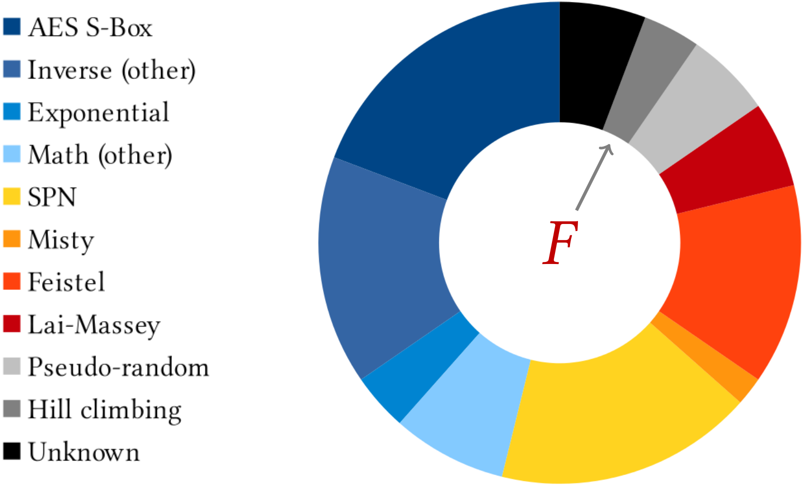
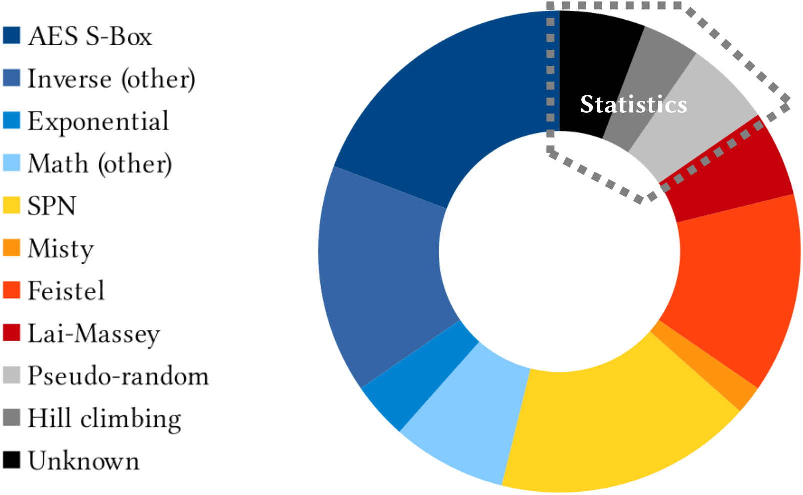


Different Techniques



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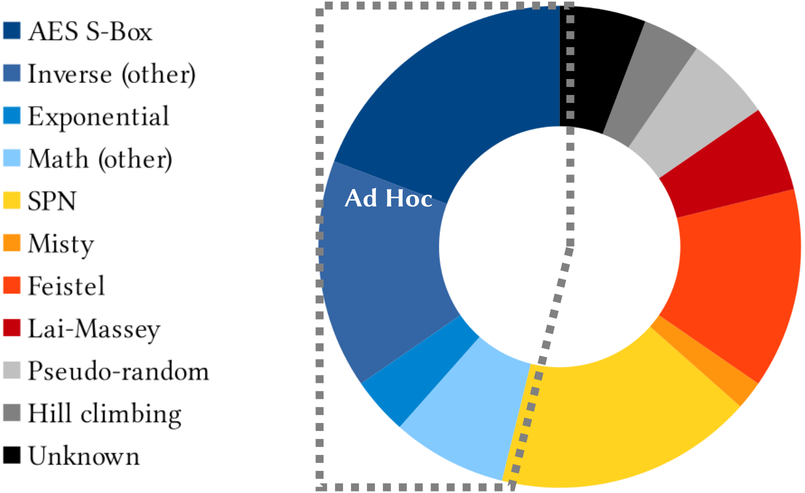

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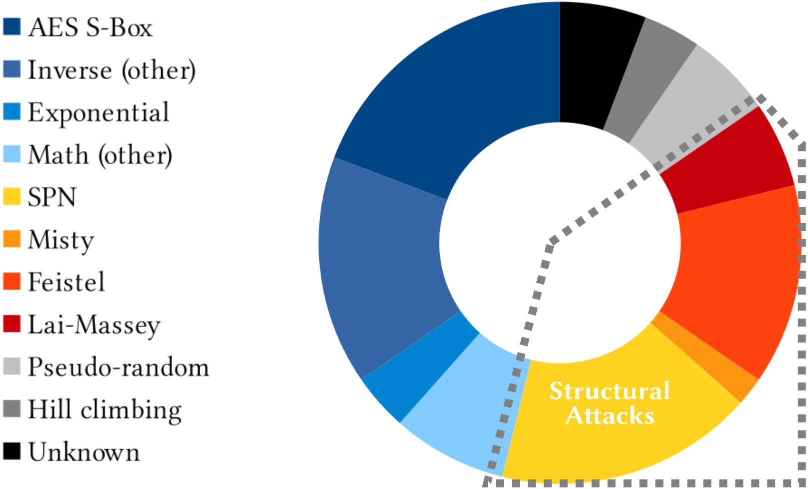
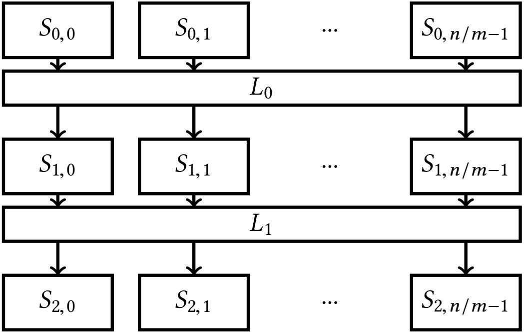


Attacks Against SPN (1/2)



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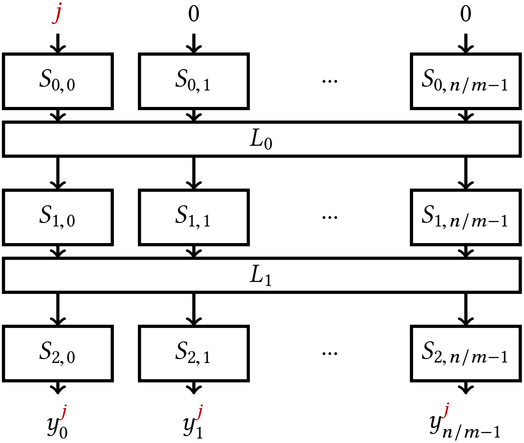

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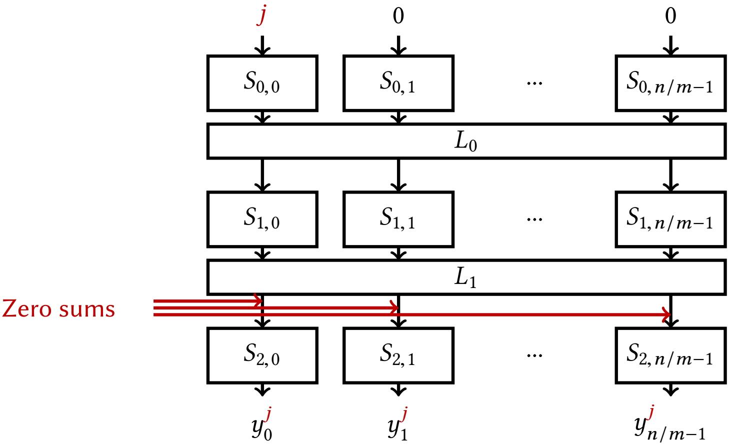

$\bigoplus_{j=0}^{2^m-1} S_{2,i}(y_i^j) = 0$, for all i.

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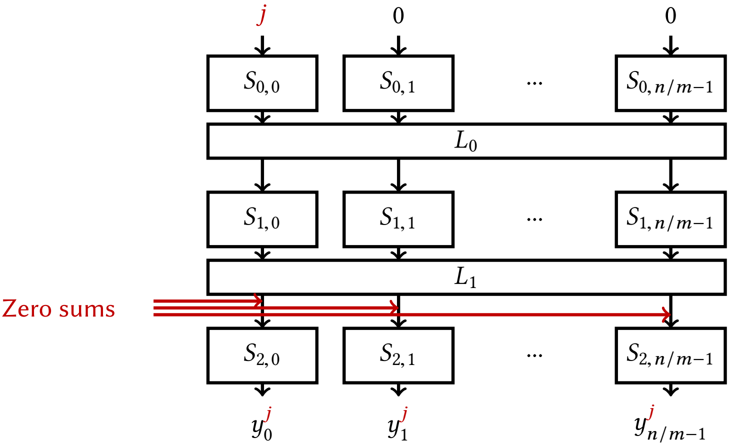
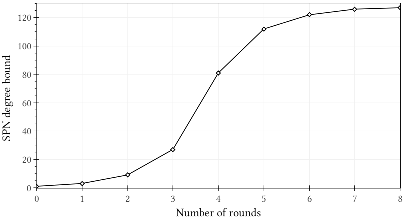


$\bigoplus_{j=0}^{2^m-1} S_{2,i}(y_i^j) = 0$, for all i. Repeat for different constant then solve system [Biryukov, Shamir, 2001]

Attacks Against SPN (2/2)

Works against more than 3 rounds if $\deg(S(AS)^{r-1})$ is low enough.



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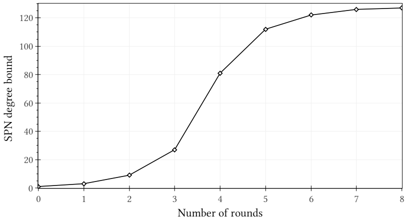


Degree Bound (SPN) [Biryukov et al., 2017]

Let σ operate on m bits, $\deg(\sigma) = m - 1$, and n be the block size.
Rhoughly speaking, $\deg\left(S(AS)^{r-1}\right) < n - 1$ as long as

$$(m-1)^{\lfloor r/2 \rfloor} < n \, .$$

Attacks Against Feistel Networks

Degree Bound (Feistel Network) [Perrin and Udovenko, 2016]

Let $\{F_i\}_{i<r}$ be permutations of $\mathbb{F}_2^{n/2}$ of degree d and let $\mathcal{F}^r(F)$ denote the r-round n-bit Feistel Network with round function F_i. If

$$d^{\lfloor r/2 \rfloor - 1} + d^{\lceil r/2 \rceil - 1} < n \, ,$$

then some degree $n-1$ terms in the ANF of $\mathcal{F}^r(F)$ are missing.

What Does it Take to Have Full Degree?

The degree based distinguishers for SPNs and Feistel networks can be seen as particular cases of this lemma.

Lemma

Let $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$ be a Boolean function and let $G : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$ be a permutation. Then:

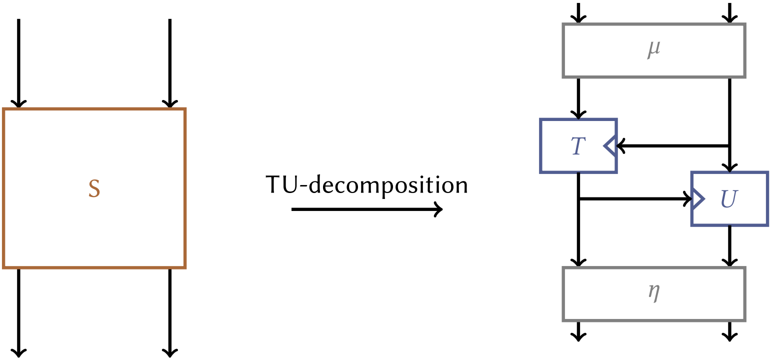
$$\deg(F \circ G) = n - 1 \implies \deg(F) + \deg(G^{-1}) \geq n \ .$$

Outline

Plan of this Section

What is the TU-Decomposition?

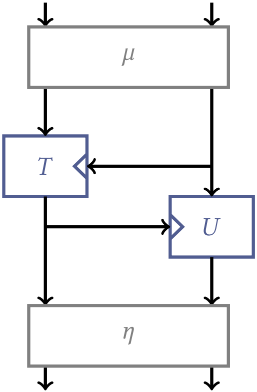
The *TU-decomposition* is a decomposition algorithm working against vast groups of algorithms: 3-round Feistel, Dillon's APN permutation, SAS, ...



T and U are mini-block ciphers ; μ and η are linear permutations.

TU-Decomposition in a Nutshell

Let $\mathcal{L}$ be the LAT of the target $S : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$.



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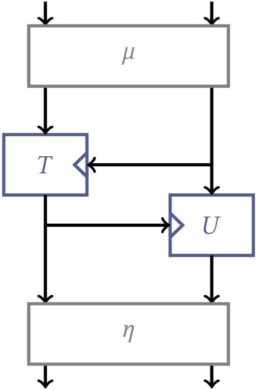
Let $\mathcal{L}$ be the LAT of the target $S : \mathbb{F}_2^n \to \mathbb{F}_2^n$.

1. Identify vector spaces $\mathcal{U}$ and $\mathcal{V}$ of dimension $n/2$ such that:

$$\mathcal{L}(a, b) = 0,\ \forall (a, b) \in \mathcal{U} \times \mathcal{V}\ .$$



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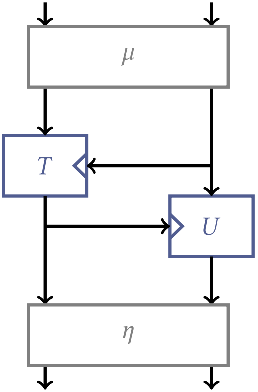
2. Deduce linear permutations μ' and η' such that

$$\mathcal{L}(\mu'(a), \eta'(b)) = 0,\ \forall (a, b) \in \mathbb{F}_2^{n/2} \times \mathbb{F}_2^{n/2}$$



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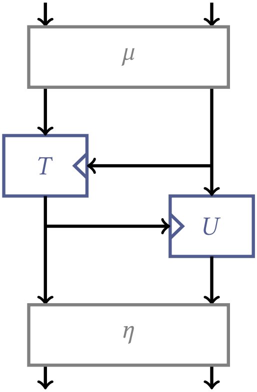
3. Built new LAT $\mathcal{L}'$ such that

$$\mathcal{L}'(a, b) = \mathcal{L}(\mu'(a), \eta'(b))$$

and recover S' with LAT $\mathcal{L}'$. Deduce μ, η.



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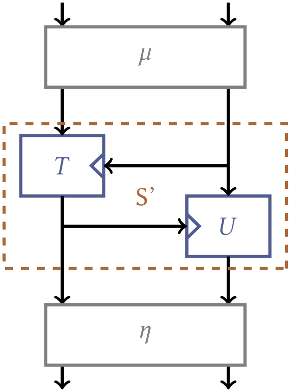


Bootstrapping TU-Decomposition

OK... But how do we find $\mathcal{U}$ and $\mathcal{V}$?

Bootstrapping TU-Decomposition

OK... But how do we find $\mathcal{U}$ and $\mathcal{V}$?

For now: we just look at the LAT and hope for the best!

Kuznyechik/Stribog

Stribog

Type Hash function

Publication [GOST, 2012]

Kuznyechik

Type Block cipher

Publication [GOST, 2015]

Kuznyechik/Stribog

Stribog

Type	Hash function
Publication	[GOST, 2012]

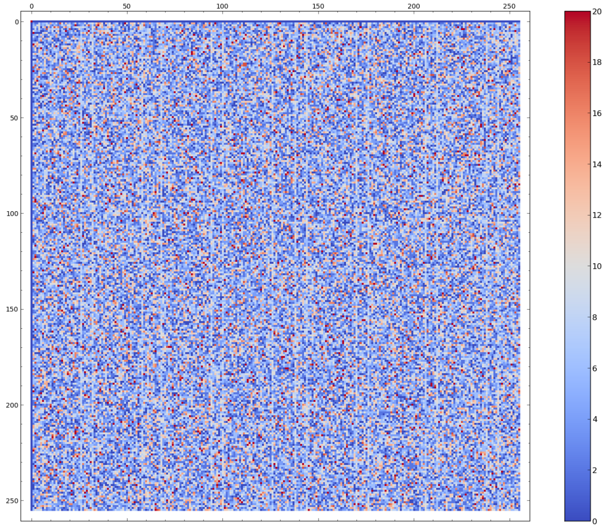
Kuznyechik

Type	Block cipher
Publication	[GOST, 2015]

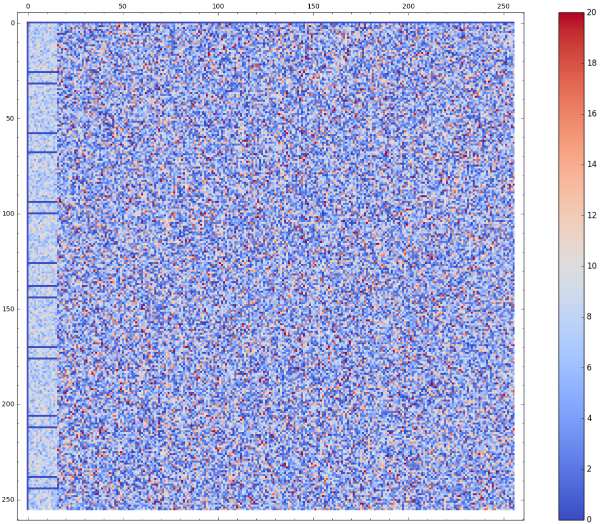
Common ground

- Both are standard symmetric primitives in Russia.
- Both were designed by the FSB (TC26).
- Both use the same 8×8 S-Box, π.

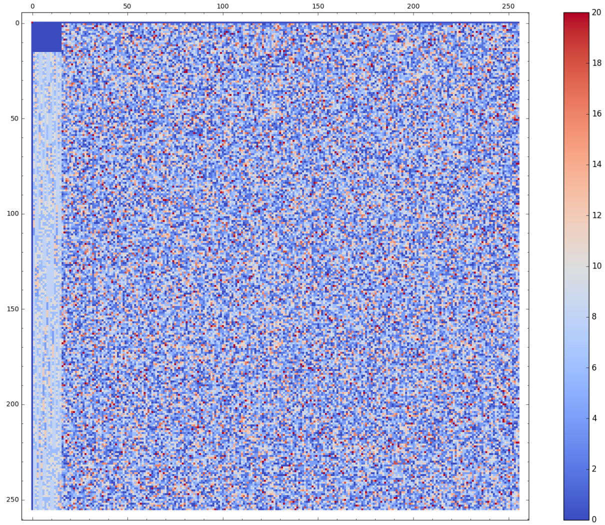
The LAT of the S-Box of Kuznyechik



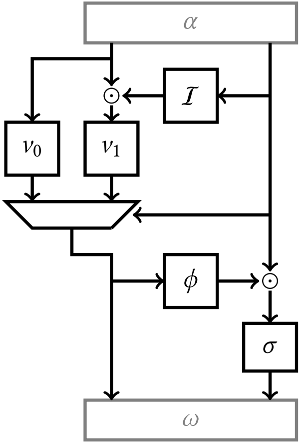
Applying one Linear Layer



Applying two Linear Layers



Final Decomposition Number 1



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α Linear permutation

$\mathcal{I}$ Inversion in $\mathbb{F}_{2^4}$

ν_0, ν_1, σ 4×4 permutations

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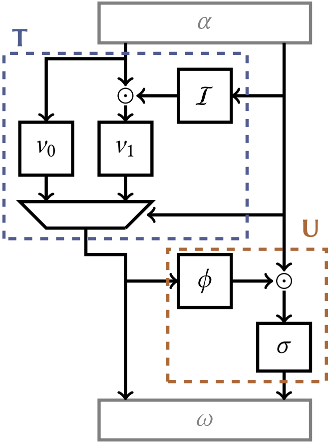

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Conclusion for Kuznyechik/Stribog?

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Conclusion for Kuznyechik/Stribog?

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... or was it?

Belarussian inspiration

- The last standard of Belarus [Bel. St. Univ., 2011] uses an 8-bit S-box,
- somewhat similar to π...

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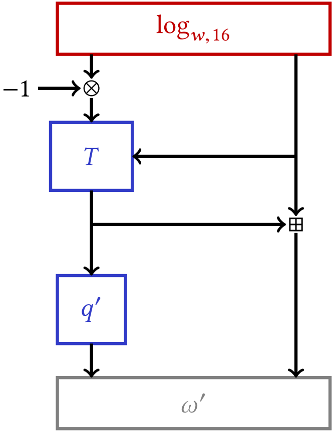
... or was it?

Belarussian inspiration

- The last standard of Belarus [Bel. St. Univ., 2011] uses an 8-bit S-box,
- somewhat similar to π...
- ... based on a finite field exponential!

Final Decomposition Number 2 (!)



	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
T_0	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
T_1	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
T_2	0	1	2	3	4	5	6	7	8	9	a	b	c	d	f	e
T_3	0	1	2	3	4	5	6	7	8	9	a	b	c	f	d	e
T_4	0	1	2	3	4	5	6	7	8	9	a	b	f	c	d	e
T_5	0	1	2	3	4	5	6	7	8	9	a	f	b	c	d	e
T_6	0	1	2	3	4	5	6	7	8	9	f	a	b	c	d	e
T_7	0	1	2	3	4	5	6	7	8	f	9	a	b	c	d	e
T_8	0	1	2	3	4	5	6	7	f	8	9	a	b	c	d	e
T_9	0	1	2	3	4	5	6	f	7	8	9	a	b	c	d	e
T_a	0	1	2	3	4	5	f	6	7	8	9	a	b	c	d	e
T_b	0	1	2	3	4	f	5	6	7	8	9	a	b	c	d	e
T_c	0	1	2	3	f	4	5	6	7	8	9	a	b	c	d	e
T_d	0	1	2	f	3	4	5	6	7	8	9	a	b	c	d	e
T_e	0	1	f	2	3	4	5	6	7	8	9	a	b	c	d	e
T_f	0	f	1	2	3	4	5	6	7	8	9	a	b	c	d	e

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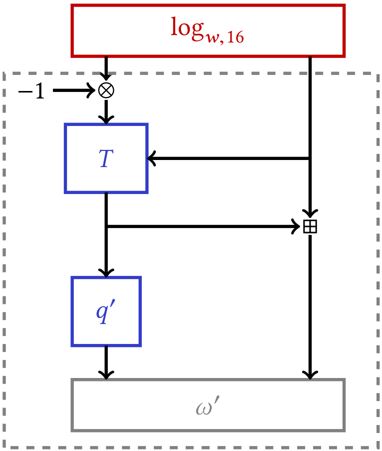

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
T_0	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
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T_3	0	1	2	3	4	5	6	7	8	9	a	b	c	f	d	e
T_4	0	1	2	3	4	5	6	7	8	9	a	b	f	c	d	e
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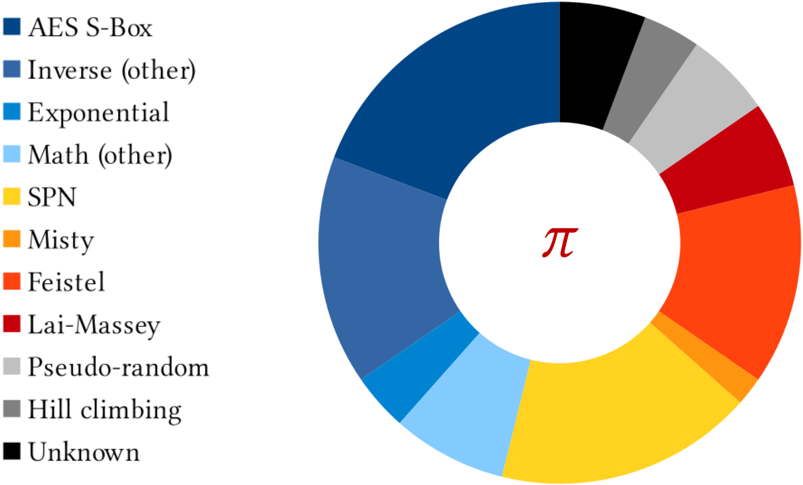
Conclusion on Kuznyechik/Stribog



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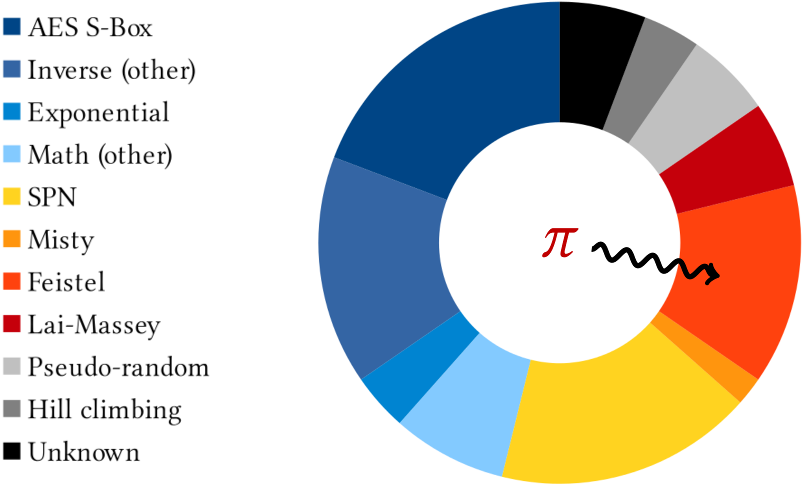

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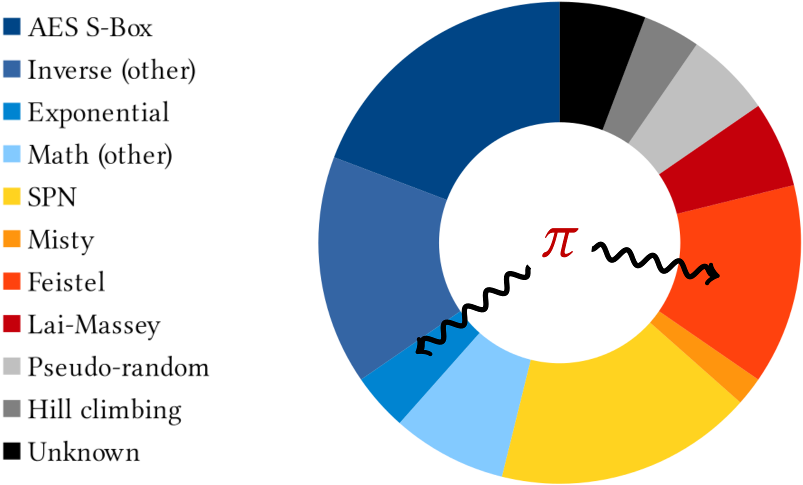

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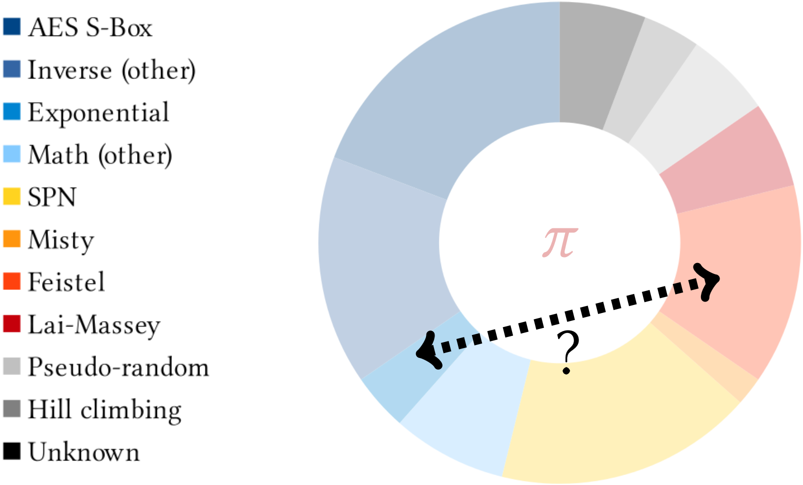

Outline

Plan of this Section

The Big APN Problem

Definition (APN function)

A function $f : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$ is Almost Perfect Non-linear (APN) if

$$f(x \oplus a) \oplus f(x) = b$$

has 0 or 2 solutions for all $a \neq 0$ and for all b.

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Big APN Problem

Are there APN permutations operating on $\mathbb{F}_2^n$ where n is even?

Dillon et al.'s Permutation

Only One Known Solution!

For $n = 6$, Dillon et al. found an APN permutation.

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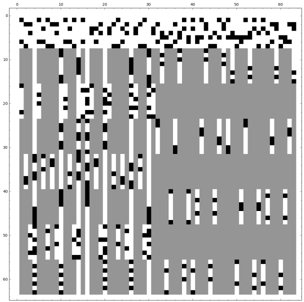

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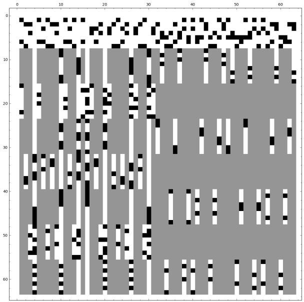

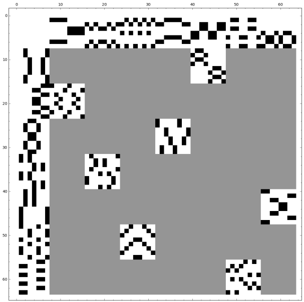

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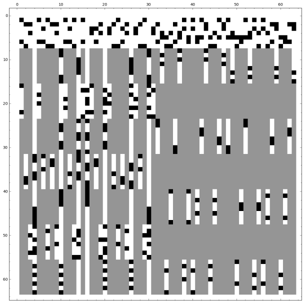
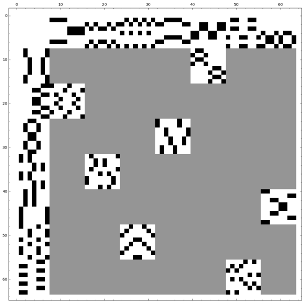
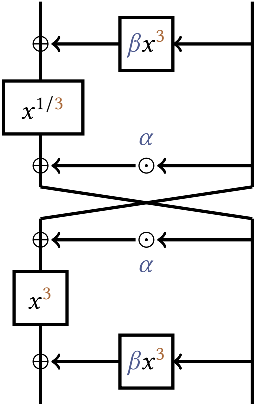




It is possible to make a TU-decomposition!

On the Butterfly Structure



Definition (Open Butterfly $H^3_{\alpha,\beta}$)

This permutation is an open butterfly.

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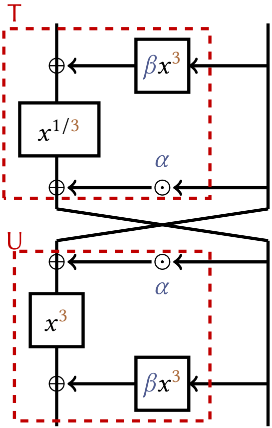

Definition (Open Butterfly $H^3_{\alpha,\beta}$)

This permutation is an open butterfly.

Lemma

Dillon's permutation is affine-equivalent to $H^3_{w,1}$, where $\mathrm{Tr}(w) = 0$.

CCZ-equivalence (1/2)

Definition (CCZ-equivalence)

Let F and G be functions of $\mathbb{F}_2^n$. They are CCZ-equivalent if there exists a linear permutation L of $\mathbb{F}_2^n \times \mathbb{F}_2^n$ such that

$$\left\{\left(x, F(x)\right), \forall x \in \mathbb{F}_2^n\right\} = \left\{L\left(x, G(x)\right), \forall x \in \mathbb{F}_2^n\right\}$$

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Properties

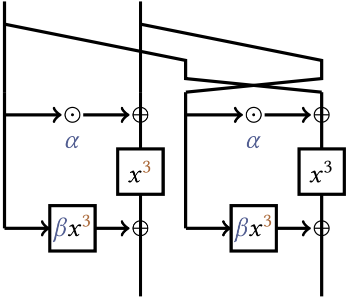
CCZ-equivalence preserves:

- the distribution of the coefficients in the LAT (Walsh spectrum),
- the distribution of the coefficients in the DDT.

It does not preserve:

- the position of the DDT/LAT coefficients
- the algebraic degree.

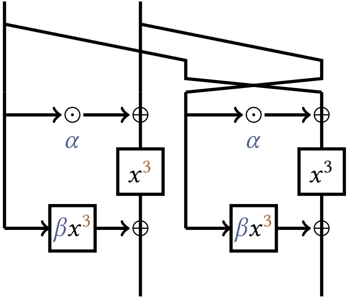
Closed Butterflies



Definition (Closed butterfly $V^3_{\alpha,\beta}$)

This quadratic function is a closed butterfly.

Closed Butterflies



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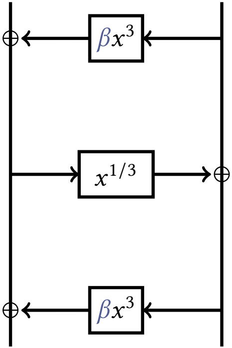
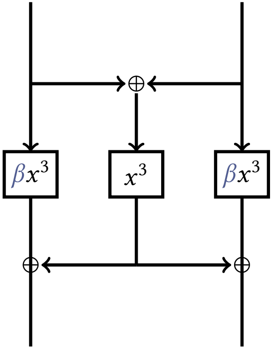
Lemma (Equivalence)

Open and closed butterflies with the same parameters are CCZ-equivalent.

Butterflies and Feistel Networks

When $\alpha = 1$, butterflies can be greatly simplified.





Some Properties of Butterflies

Theorem (Properties of butterflies [Canteaut et al., 2017])

Let $\mathsf{V}^3_{\alpha,\beta}$ *and* $\mathsf{H}^3_{\alpha,\beta}$ *be butterflies operating on* $2n$ *bits,* n *odd. Then:*

- $\deg\left(\mathsf{V}^3_{\alpha,\beta}\right) = 2$,
- *if* $n = 3$, $\mathrm{Tr}(\alpha) = 0$ *and* $\beta + \alpha^3 \in \{\alpha, 1/\alpha\}$, *then*
$$\max(DDT) = \mathbf{2},\ \max(\mathcal{W}) = 2^{n+1} \ \textit{and}\ \deg\left(\mathsf{H}^3_{\alpha,\beta}\right) = n+1\,,$$
- *if* $\beta = (1+\alpha)^3$, *then*
$$\max(DDT) = 2^{n+1},\ \max(\mathcal{W}) = 2^{(3n+1)/2} \ \textit{and}\ \deg\left(\mathsf{H}^3_{\alpha,\beta}\right) = n\,,$$
- *otherwise,*
$$\max(DDT) = 4,\ \max(\mathcal{W}) = 2^{n+1} \ \textit{and}\ \deg\left(\mathsf{H}^3_{\alpha,\beta}\right) \in \{n, n+1\}$$
and $\deg\left(\mathsf{H}^3_{\alpha,\beta}\right) = n$ *if and only if*
$$1 + \alpha\beta + \alpha^4 = (\beta + \alpha + \alpha^3)^2\,.$$

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Plan of this Section

Conclusion

- We can recover the majority of known S-Box structures and derive new results about Skipjack and Kuznyechik.

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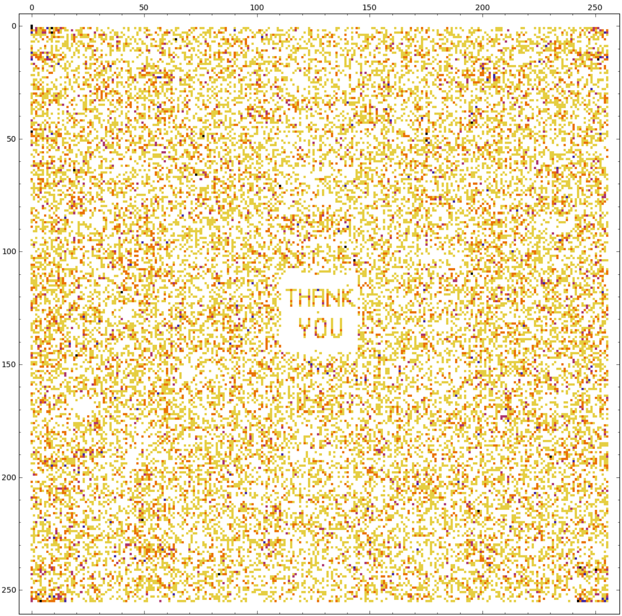
Conclusion

- We can recover the majority of known S-Box structures and derive new results about Skipjack and Kuznyechik.
- We can generalize the permutation of Dillon et al...
- but we can prove that our generalizations are never APN (except in the known case).
- There are still S-Boxes with unknown building strategies (CMEA, CSS)!

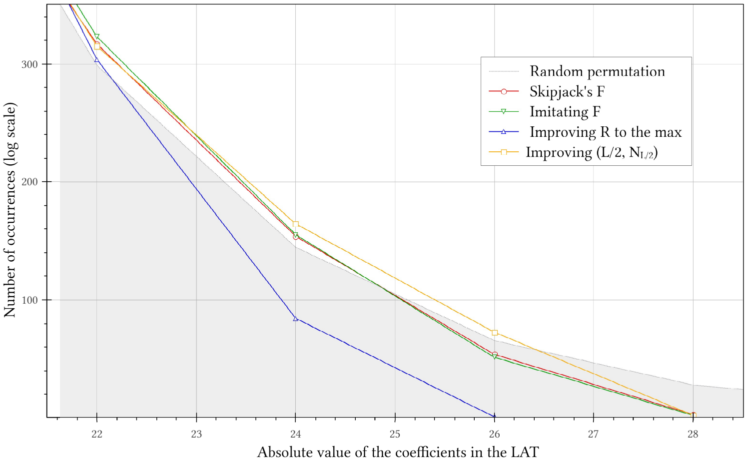
The Last S-Box

14	11	60	6d	e9	10	e3	2	b	90	d	17	c5	b0	9f	c5
d8	da	be	22	8	f3	4	a9	fe	f3	f5	fc	bc	30	be	26
bb	88	85	46	f4	2e	e	fd	76	fe	b0	11	4e	de	35	bb
30	4b	30	d6	dd	df	df	d4	90	7a	d8	8c	6a	89	30	39
e9	1	da	d2	85	87	d3	d4	ba	2b	d4	9f	9c	38	8c	55
d3	86	bb	db	ec	e0	46	48	bf	46	1b	1c	d7	d9	1b	e0
23	d4	d7	7f	16	3f	3	3	44	c3	59	10	2a	da	ed	e9
8e	d8	d1	db	cb	cb	c3	c7	38	22	34	3d	db	85	23	7c
24	d1	d8	2e	fc	44	8	38	c8	c7	39	4c	5f	56	2a	cf
d0	e9	d2	68	e4	e3	e9	13	e2	c	97	e4	60	29	d7	9b
d9	16	24	94	b3	e3	4c	4c	4f	39	e0	4b	bc	2c	d3	94
81	96	93	84	91	d0	2e	d6	d2	2b	78	ef	d6	9e	7b	72
ad	c4	68	92	7a	d2	5	2b	1e	d0	dc	b1	22	3f	c3	c3
88	b1	8d	b5	e3	4e	d7	81	3	15	17	25	4e	65	88	4e
e4	3b	81	81	fa	1	1d	4	22	0	6	1	27	68	27	2e
3b	83	c7	cc	25	9b	d8	d5	1c	1f	e5	59	7f	3f	3f	ef

Conclusion



Details About Skipjack



Proof of Full Degree Condition

If $\deg(F \circ G) = n - 1$, then $\exists i \leq n$ such that $\bigoplus_{x \in C_i} (F \circ G)(x) = 1$.

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I_i is affine ($I_i(x) = 1 + x_i$). Thus, the sum can be equal to 1 only if

$$\deg(F) + \deg(G^{-1}) \geq n\,.$$

$\square$

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